error for a given $\gamma(\mathbf{a})$ tends to zero as $\mathbf{a}\rightarrow\infty$ whereas the error limit converges to a nonvanishing constant. Th accuracy of these limits is difficult to assess and moreover, since formula (22) holds for all values of $\mathbf{a}$, if a given $\gamma(\mathbf{a})$ is known to be a good approximation to $\gamma_0(\mathbf{a})$ in some regions of space, the actual error in this region may be expected to be much less than that given by inequality (22). If $\gamma(\mathbf{a})$ is obtained through the variation principle, it is likely to be accurate near the singularities of the potential and, consequently, in such a case, formula (22) is likely to overestimate considerably the error near the nuclei. All this, however, does not impair our general conclusions as stated in the introduction.

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Regge Trajectories and Elementary Poles*

REINHARD OEHME
The Enrico Fermi Institute for Nuclear Studies, and the Department of Physics, The University of Chicago, Chicago, Illinois
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Elementary poles are introduced as poles of physical partial wave amplitudes $F_l(s)$ which are not present in the analytic interpolating function $F(s,\lambda)$. It is shown that $F_l(s)=F(s,l)$ for $l>1$, and that all particles with spin larger than one must be members of Regge trajectories; only bosons are considered explicitly. Additional restrictions are discussed which would make it possible to eliminate elementary poles also for spin one and zero. The possibility that the physical s-wave amplitude $F_0(s)$ is not determined by the interpolation function $F_+(s,0)$ could be used to avoid the ghost associated with the vacuum trajectory. The problem of branch-point trajectories is discussed briefly.

1. INTRODUCTION

THE possibility of describing particles as members of Regge trajectories[1,2] in the complex angular momentum plane raises the important question whether all particles should be represented by these moving poles. As an alternative, we can envisage a description of certain particles and resonances in terms of "elementary poles"[3] of the relevant amplitudes, where these poles are not related to the moving poles of the interpolating partial wave function at all. The two ways of representing particles suggest a possibility for making a qualitative distinction between "elementary" and "composite" particles. Although one may dislike such a distinction on "philosophical" grounds, it is of interest to see to what extent it may be excluded on the basis of the general notions of relativistic dispersion theory as well as specific experimental information. In this note we will be concerned mainly with strongly interacting particles, but the two ways of describing particles may also be of interest in connection with the distinction between strong and weak interactions (and perhaps electromagnetic interactions).

In an earlier publication[3] we have already described how elementary poles can be present in a physical partial wave amplitude $F_l(s)$ such that these poles are not related to a singularity of the interpolating function $F(s,\lambda)$.[4] We have also given an argument showing that $F(s,l)=F_l(s)$ for $l>1$, and that all particles with spin larger than one must be members of pole trajectories in the λ plane. However, this argument depends upon the existence of a Sommerfeld-Watson representation of the invariant amplitude $F(s,t)$, or a related representation which is valid for $t\rightarrow\infty$ and for some interval on the negative s axis or around $s=0$.

It is the purpose of this paper to give a more detailed description of elementary poles as compared to Regge poles within the framework of relativistic dispersion theory, and to give a more general proof for the fact that particles with spin larger than one must be manifestations of angular momentum trajectories. Furthermore, we show that any information to the effect that the high-energy limit $(t,u\rightarrow\infty)$ of the invariant ampli-

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[1] T. Regge, Nuovo Cimento **18**, 947 (1960).
[2] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961) and **8**, 41 (1962); R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962); G. F. Chew, S. C. Frautschi, and S. Mandelstam, *ibid.* **126**, 1204 (1962); S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, *ibid.* **126**, 2204 (1962).
[3] R. Oehme, Phys. Rev. Letters **9**, 358 (1962).
[4] We define here elementary poles independent of perturbation theory. For a definition within the framework of perturbation theory, see S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962); this paper contains further references.

tude $F(s,t)$ stays below the power law t^1 (or t^0) and u^1 (or u^0) for any small interval in s is sufficient to prove that particles with spin one (or with spin zero and spin one) must be described by moving poles. If one is willing to assume the existence of some form of the Sommerfeld-Watson representation for $t\to\infty$ and some $s\leq 0$, then it follows[3] that an elementary pole describing a spin-one or a spin-zero particle always gives rise to a term $C_1(s)t^1$ or $C_0(s)t^0$ in the asymptotic expansion of $F(s,t)$,where $C_1(s)$ and $C_0(s)$ are analytic functions which cannot vanish in a whole neighborhood.

2. ANALYTIC INTERPOLATION OF PARTIAL-WAVE AMPLITUDES

In order to simplify the discussion, we consider the scattering of spin-zero particles with equal mass μ, but we do not consider them as identical particles. In a channel where s is the energy variable, the invariant amplitude $F(s,t)$ is related to the c.m. amplitude by

$$T_{\text{c.m.}}(s,t)=(2/\sqrt{s})F(s,t). \tag{1}$$

We define the physical partial-wave amplitude by

$$F_l(s)=\frac{1}{2}\int_{-1}^{+1} d\cos\vartheta\, P_l(\cos\vartheta)F(s,t(\cos\vartheta)) \tag{2}$$

so that

$$F(s,t)=\sum_{l=0}^{\infty}(2l+1)F_l(s)P_l\left(1+\frac{t}{2q^2(s)}\right), \tag{3}$$

where $4q^2(s)=s-4\mu^2$. The existence of a dispersion relation for $F(s,t)$ at fixed values of s with a finite number of subtractions implies that there is a number N such that for $l>N$

$$F_l(s)=\frac{1}{\pi}\int_{v_0}^{\infty} dv\,\frac{1}{2q^2}Q_l\left(1+\frac{v}{2q^2}\right)\times[A_t(s,v)+(-1)^lA_u(s,v)]. \tag{4}$$

Here $A_t(s,t)$ and $A_u(s,u)$ are the absorptive parts of $F(s,t)$ in the t and the u channel, respectively, and $v_0>0$ is sufficiently small for the integral to include possible single-particle contributions in these channels. We can also define the interpolating function

$$F_\pm(s,\lambda)=\frac{1}{\pi}\int_{v_0}^{\infty} dv\,\frac{1}{2q^2}Q_\lambda\left(1+\frac{v}{2q^2}\right)A_\pm(s,v), \tag{5}$$

where

$$A_\pm(s,v)=A_t(s,v)\pm A_u(s,v). \tag{6}$$

The expression (5) represents an analytic function of the two complex variables λ and s for $\text{Re}\lambda>N$ and s in the cut plane. It is a unique interpolation of the partial-wave amplitudes $F_l(s)$ such that for[5] $l>N$

$$F_\pm(s,\,\lambda=l)=F_l(s)\text{ for }l=\text{even/odd}. \tag{7}$$

[5] See, for example, V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **41**, 1962 (1961) [translation: Soviet Phys.—JETP **14**, 1395 (1962)].

Let us now assume that there is no natural boundary in the λ plane, at least for $\text{Re}\lambda>-\frac{1}{2}$. Then we can continue the function $F_\pm(s,\lambda)$ into the region $\text{Re}\lambda<N$, and for $\lambda=l$, $l=$even/odd it uniquely defines amplitudes $F_\pm(s,l)$. However, *a priori* these amplitudes are *not* identical to the actual partial wave amplitudes $F_l(s)$ as defined in Eq. (2). We note that neither the representation (5) for $F_\pm(s,\lambda)$ nor the expression (4) for $F_l(s)$ is known to be valid for $l<N$. However, for real $s\leq 0$ we have an asymptotic bound on $F(s,t)$ for $t\to\infty$ and $u\to\infty$ which is given by[6]

$$\lim_{t\to\infty}|F(s,t)|\leq\text{const }t(\ln t)^2, \tag{8}$$

and a corresponding one for $u\to\infty$. Hence the representation (4) of $F_l(s)$ is valid for $l>1$ and real $s\leq 0$ and coincides with the definition (2) of the partial wave amplitude. From the double dispersion relation for $F(s,t)$ we know that the $F_l(s)$ are always real analytic functions of s except for poles and branch points, and in this way the functions $F_l(s)$ are defined also for real $s\leq 0$.

In the domain $\text{Re}\lambda>N$, the function $F_\pm(s,\lambda)$ is defined by the representation (5) as a regular function in λ. As a function of s it has singularities at the same points as the $F_l(s)$ for $l>N$. The position of these singularities is independent of λ, but their character may depend upon λ. If we now continue $F_\pm(s,\lambda)$ into the region $\text{Re}\lambda\leq N$, we will encounter s-dependent singularities in the λ plane which correspond to singular surfaces $\lambda=f(s)$, where $f(s)$ is a real analytic function. These surfaces manifest themselves in the s plane as moving singularities $s=\varphi(\lambda)$. As $\text{Re}\lambda$ increases, the singular points $s=\varphi(\lambda)$ disappear from the physical sheet, poles through the two-particle threshold, branch points (if they exist at all) through many-particle thresholds, etc.[7] There cannot be any nonmoving singularities in the s plane for $\text{Re}\lambda\leq N$ except those which are also present for $\text{Re}\lambda>N$, because a fixed singular point would have to disappear suddenly as $\text{Re}\lambda$ increases above N, and such behavior for $F_\pm(s,\lambda)$ is not possible. This result follows from the continuity theorem for analytic functions of two or more complex variables; it has been obtained first by Oehme and Tiktopoulos.[8]

After this brief survey of the analytic properties of $F_\pm(s,\lambda)$, we consider now the representation (5) for real $s\leq 0$. Just as in the case of integer l, it follows from the bound (8) that $A_\pm(s,v)$ is bounded by $v(\ln v)^2$ for $v\to\infty$ and consequently, because of $Q_\lambda(1+v/2q^2)\sim v^{-\lambda-1}$, the function $F_\pm(s,\lambda)$ is regular for $\text{Re }\lambda>1$ for all s on the negative real axis. Since both representations (4) and (5) are valid for real $s\leq 0$, and because (4) actually repre-

[6] M. Froissart, Phys. Rev. **123**, 1053 (1961).
[7] R. Oehme, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 564; J. L. Challifour and R. J. Eden, (unpublished); V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **42**, 1260 (1962) [translation: Soviet Phys.—JETP **15**, 873 (1962)].
[8] R. Oehme and G. Tiktopoulos, Phys. Letters **2**, 86 (1962).

sents the physical amplitude (2), it follows that Eq. (7) is valid for $l>1$ and $s\leq 0$. But the equality of $F_\pm(s,l)$ and $F_l(s)$ in a real neighborhood is sufficient to guarantee that $F_\pm(s,l)=F_l(s)$, $l=$even/odd, $l>1$ for all s. Even though these amplitudes have branch points on the negative real s axis, we can always find an interval where they can be analytically contined through the cut into another Riemann sheet, and such an interval may serve as a real neighborhood.

3. MOVING POLES

In the previous section we have seen that the interpolating function $F_\pm(s,\lambda)$, which is uniquely determined by the partial wave amplitudes $F_l(s)$ with l larger than some finite integer N, actually coincides with the physical amplitudes for all $\lambda=l>1$. Consequently the invariant amplitude $F(s,t)$ is determined by the function $F_\pm(s,\lambda)$ apart from an expression of the form

$$\Delta F(s,t)=[F_0(s)-F_+(s,0)]+[F_1(s)-F_-(s,1)]P_1(1+t/2q^2)=a(s)+b(s)t. \quad (9)$$

To what extent does this result imply that all stable particles and resonances with spin larger than one are manifestations of angular momentum trajectories corresponding to moving singularities $s=\varphi(\lambda)$ in the physical s plane or in a secondary Riemann sheet? We have seen that $F_\pm(s,\lambda)$ has no λ-independent singularities in s which are not also present in the limit of large Re λ.[8] This result applies to single-particle poles in the physical sheet of the s plane as well as to resonance poles in a secondary sheet. For instance, the poles $F_\pm(s,\lambda)$ in the sheet reached through the cut $4\mu^2\leq s<s_i$ (e.g., $s_i=16\mu^2$ for $\pi\pi$ scattering) are due to zeros of

$$S_\pm(s,\lambda)=1+2i\rho(s)F_\pm(s,\lambda), \quad (10)$$

where

$$\rho(s)=[(s-4\mu^2)/s]^{1/2},$$

We see that any λ-independent pole of $F_\pm(s,\lambda)$ would be present in all physical amplitudes $F_l(s)$ for $l>1$. It would correspond to a situation where we have an infinite number of particles with the same mass and the spin values $l=2, 3, \cdots$. This is absurd, at least as far as stable particles and measurable resonances are concerned; hence we conclude that all particles with $l>1$ must be members of trajectories $s=\varphi(\lambda)$ or $\lambda=\alpha(s)$.

The next question is whether these trajectories describing particles and resonances are simple pole trajectories $\lambda=\alpha(s)$ such that

$$F(s,\lambda)\sim R(s)/[\alpha(s)-\lambda]+\cdots, \quad (11)$$

for $\lambda\rightarrow\alpha(s)$. As an alternative, we may think of a pole term with a superimposed branch point of varying character which reduces to a simple pole for $\lambda=l$, like, for instance,

$$\frac{R(s)}{[\lambda-\alpha(s)]^{1+\beta(s)}}, \quad (12)$$

with $\alpha(s=m^2(l))=l$, $\beta(s=m^2(l))=0$, and $\mathrm{Re}\beta(s)>-1$. However, by expanding $\alpha(s)$ and $\beta(s)$ around $s=m^2(l)$, we find an expression of the form

$$[m^2(l)-s]^{-1}\exp\{[m^2(l)-s]\ln[m^2(l)-s]\},$$

which is *not* an isolated pole in the s plane. Also, a singularity of $F(s,\lambda)$ like (12), which is not a simple pole, cannot disappear from the physical sheet of the s plane through an elastic threshold. We conclude, therefore, that the particles with $l>1$ should be described by simple moving poles.

So far we have only discussed the single-particle singularities, but it should not be forgotten that the unitarity condition requires the existence of many particle states which give rise to branch lines of $F_l(s)$ and $F_\pm(s,\lambda)$ like $s\geq 4m^2(l)$, $s\geq[m(l)+\mu]^2$, etc. These cuts will be discussed in a later section.

4. ELEMENTARY POLES

Particles and resonances with spin zero and one may, of course, also be described by pole trajectories in the λ plane, but on the basis of the bound (8) alone we cannot exclude the possibility that the partial wave amplitudes $F_0(s)$ and $F_1(s)$ have poles which are not present in $F_\pm(s,\lambda)$ and vice versa. We call these poles of $F_{0,1}(s)$ "elementary poles" in distinction from the moving poles or Regge poles of the interpolating function $F_\pm(s,\lambda)$. Because of the possible existence of these elementary poles, it is of special interest to study the functions

$$\Delta F_0(s)=F_0(s)-F_+(s,0),$$
$$\Delta F_1(s)=F_1(s)-F_-(s,1). \quad (13)$$

In the physical sheet of the s plane they are regular functions except for poles and cuts on the real axis. They have, however, no left-hand branch points. In order to prove this we consider the representation (5) for $F_\pm(s,\lambda)$, $\mathrm{Re}\lambda>N$ and compute the discontinuity for real $s\leq 0$. From the dispersion relation of the absorptive part $A_\pm(s,v)$ and the well-known discontinuity of $Q_\lambda(z)$ we find

$$\frac{1}{2i}\mathrm{disc}[q^{-2\lambda}(s+i0)F_\pm(s+i0,\lambda)]_{s\leq 0}$$
$$=\frac{1}{2}\int_{4\mu^2}^{4\mu^2-s}dv\frac{1}{2q^2(-q^2)^\lambda}P_\lambda\left(-1-\frac{v}{2q^2}\right)A_\pm(s-i0,v)$$
$$+\frac{1}{\pi}\int_{v_-(s)}^{v_+(s)}dv\frac{1}{2q^2(-q^2)^\lambda}Q_\lambda\left(-1-\frac{v}{2q^2(s+i0)}\right)$$
$$\times[\rho_{tu}(4\mu^2-s-v,v)\pm\rho_{tu}(v,4\mu^2-s-v)], \quad (14)$$

where

$$v_\pm(s)=\tfrac{1}{2}(4\mu^2-s)\pm\tfrac{1}{2}[s(s+8\mu^2)]^{1/2}, \quad (15)$$

and the second term in Eq. (14) contributes only for

$s\leq -8\mu^2$. For reasons of simplicity, we have taken $v_0=4\mu^2$ corresponding to $\pi\pi$ scattering. Since in Eq. (14) the limits of integration are finite, it represents the discontinuity as a meromorphic function of λ which, in the finite plane, has only the poles of Q_λ at $\lambda=-n$, $n=1, 2, \cdots$. The corresponding discontinuity of $F_l(s)$ can be obtained from Eq. (4). The resulting representations are valid for all l and we find

$$\text{disc } \Delta F_{0,1}(s+i0)|_{s\leq 0}=0. \tag{16}$$

Note that the second term in Eq. (14) plus the contribution from Im $A_\pm(s-i0, v)$ in the first term vanish for $\lambda=l$ and even/odd values of l.

We see that the differences $\Delta F_{0,1}$ are analytic except for poles and cuts along the positive real axis. Suppose there is an interval $4\mu^2\leq s<s_i$ where $F(s,t)$ satisfies the elastic unitarity condition. Then we also have the continued unitarity equation

$$\text{disc}F_\pm(s+i0, \lambda)=2i\rho(s+i0)F_\pm(s+i0, \lambda)F_\pm(s-i0, \lambda)$$

for $4\mu^2\leq s<s_i$, and hence in this region the difference (13) can be written in the form

$$\Delta F_{0,1}(s)=\rho(s)[\sin\delta_{0,1}(s)e^{i\delta_{0,1}(s)} -\sin\delta(s;0,1)e^{i\delta(s;0,1)}], \tag{17}$$

with real phase shifts $\delta_l(s)$ and $\delta(s,l)$. From Eq. (17) we see that $\Delta F_{0,1}(s)$ cannot be just a pole term like

$$\Delta F_0(s)=g^2m^2/(m^2-s),$$

because then Im $\Delta F_0(s+i0)=0$. This implies $\delta_0(s) =\pm\delta(s,0)$ and hence $F_0(s)=\pm F(s,0)$. The lower sign is absurd because it leads to Im $F_0=0$ for $4\mu^2<s<s_i$; we, therefore, find that $g^2=0$. Hence $\Delta F_{0,1}(s)$ must be of the form

$$\Delta F_{0,1}(s)=\frac{1}{\pi}\int_{s_0}^{\infty} ds' \frac{\text{Im}\Delta F_{0,1}(s'+i0)}{s'-s}, \tag{18}$$

where we have ignored a possible s-wave subtraction and where $s_0>0$ is sufficiently small to take into account single-particle poles.

It is of interest to note that the branch points associated with an elementary pole of $F_{0,1}(s)$ must also be present in $F_\pm(s,\lambda)$, although the pole itself is not present in $F_+(s,0)$ or $F_-(s,1)$. Suppose $F_0(s)$ has a pole at $s=m^2<4\mu^2$. Then all the partial wave amplitudes $F_l(s)$ have branch lines $s\geq(m+\mu)^2$, $s\geq 4m^2$, etc., and since for $l>1$, $F_\pm(s,l)=F_l(s)$, $l=$even/odd, the interpolating function $F_\pm(s,\lambda)$ must also have these branch lines. Of course, the weights along the cuts may be different for $F_0(s)$ and $F_+(s,0)$.

The existence of branch points in $F_\pm(s,\lambda)$ which are associated with elementary poles already shows that these pole terms are not simply free parameters; they are, in fact, intimately related to other parameters of the amplitude $F(s,t)$. For instance, we can easily see that the interpolating functions $G_\pm(t,\lambda)$ of the partial wave amplitudes with $l>1$ in the t channel determine $F(s,t)$ up to an expression of the form

$$[G_0(t)-G_+(t,0)]+[G_1(t)-G_-(t,1)] \times P_1(1+u/2q^2(t))=c(t)+d(t)s, \tag{19}$$

where $u=4\mu^2-s-t$ is the momentum transfer in the t channel. Comparing Eqs. (19) and (9), we see that $G_\pm(t,\lambda)$ *determines* the poles and branch points of $\Delta F_{0,1}(s)$.

5. BRANCH POINTS

In the same way as elementary poles generate branch lines describing many-particle states which involve one or more of the particles represented by such poles, we expect that particles described by moving poles give also rise to branch lines. The essential difference between moving poles and elementary poles is not apparent in the dispersion relations for the amplitude $F(s,t)$ itself, but only when we consider the behavior of these poles as a function of the angular momentum variable λ. The variation of λ tests the behavior of the particle mass under changes of the strength of the centrifugal potential. The situation is quite different for branch points, because even though one or more Regge-type particles are present in a state with two or more particles, the continuous variation of the total angular momentum may well be completely taken up by the orbital angular momentum which is due to the relative motion of the particles. If this is the case, then the position of the corresponding branch point of $F_\pm(s,\lambda)$ in the s plane is independent of λ and only its character changes with the angular momentum variable. For instance, a branch point due to two spin-zero particles with mass $m(0)$ and μ gives rise to a term proportional to $k^{2\lambda+1}(s)$, where $k(s)$ is the momentum in the c.m. system of both particles. Even though $m^2(0)$ is a member of a family of poles $s=m^2(\lambda)$ of $F_+(s,\lambda)$, say, the branch point remains at $s=[m(0)+\mu]^2$ for all λ is the continuous variation is completely taken up by the orbital angular momentum. Evidently the complete amplitude $F(s,t)$ has a branch point at $s=[m(0)+\mu]^2$ and so have all physical partial wave amplitudes $F_l(s)$. In the representation (4) of $F_l(s)$, this branch point is contained in the absorptive parts $A_{t,u}(s,v)$ for $v\to\infty$. But then it must also be present in $F_\pm(s,\lambda)$ on the basis of the corresponding formula (5) for Re$\lambda>N$, and, as a λ-independent singularity in the s plane, it certainly will not vanish suddenly as we continue to Re$\lambda<N$. We conclude, therefore, that the interpolating function $F_\pm(s,\lambda)$ has all the branch points required by the unitarity condition for the amplitudes $F_l(s)$ as nonmoving singularities, whether they correspond to intermediate states involving Regge-type particles or particles described by elementary poles. The only question is whether, in the first case, there can be moving branch points in addition to those discussed above. On a previous occasion we have already pointed out[8,9] how

[9] R. Oehme, reference 7, and Nuovo Cimento (to be published).

such moving branch points could, in principle, be generated by Regge-pole correlations in intermediate states with three or more particles. They would, of course, give rise to branch point trajectories in the complex angular momentum plane. It is clear that these trajectories can only be associated with inelastic intermediate states of the amplitude in the s channel, because, as we have pointed out before, a moving branch point $s=s_c(\lambda)$ in the physical sheet of the s plane must disappear from the physical sheet for sufficiently large values of Re λ, and it cannot do so through an elastic branch point.[7] [Moving singularities can never disappear through left-hand branch points of $F(s,\lambda)$.[7,8]] For example, take the continuation of $F_{\pm}(s,\lambda)$ through the elastic cut $4\mu^2\leq s<s_i$; we have[10]

$$F_{\pm}^{\mathrm{II}}(s,\lambda)=F_{\pm}(s,\lambda)/[1+2i\rho(s)F_{\pm}(s,\lambda)], \qquad (20)$$

and if $F_{\pm}^{\mathrm{II}}(s,\lambda)$ has a branch point at $s=s_c(\lambda)$, then also $F_{\pm}(s,\lambda)$ has one. This argument can be generalized to other two-particle channels with particles of fixed spin.

We do not want to pursue here the question of the actual existence of the moving branch points. Their appearance is perhaps unlikely in view of the fact that $F_{\pm}(s,\lambda)$ must have branch points at all the physical thresholds like $s=[m(l)+\mu]^2$, etc., for all values of λ, these thresholds being, of course, in secondary Riemann sheets if they involve unstable particles. However, there may be causes for the existence of moving branch points other than those discussed here. From the discussion in reference 9 we find that, at least as far as three-particle intermediate states with two-particle correlations are concerned, the continuation of the inelastic unitarity condition which leads to a unique interpolating function $F_{\pm}(s,\lambda)$ seems to be the one where the spin of the correlated pair remains quantized. This implies that the continuous angular momentum variation is taken up by the kinetic angular momentum of the third particle relative to the correlated system.

6. CONCLUDING REMARKS

In Sec. 2 we used the Froissart bound for $F(s,t)$ in the t and the u channel in order to show that all particles with spin larger than one must be described by pole trajectories in the complex λ plane, but we could not exclude the possible existence of elementary poles of $F(s,t)$ with $l=0$, 1 which are not present in $F_{\pm}(s,\lambda)$. We may ask whether a further restriction of elementary poles is possible on the basis of additional information about the high-energy limits of the amplitude. For instance, suppose we know from experiments that for some interval on the negative s axis the asymptotic form of $F(s,t)$ for t, $u\rightarrow\infty$ behaves better than t, u to the first power. Then we can use our method in order to prove that $F_1(s)=F_{-}(s,1)$, and hence there can be no elementary poles with $l=1$. In the same way we can exclude elementary poles with spin zero and spin one if we know that $F(s,t)$ vanishes for t, $u\rightarrow\infty$ and some interval in s.

Present experiments[11] on high-energy p-p scattering indicate that the corresponding amplitude vanishes faster than t^1 and perhaps even t^0 for larger negative values of the momentum transfer variable s. If the same is true for p-$\bar{p}$-scattering, and if these experiments really determine the leading term in the asymptotic expansion of $F(s,t)$, then we could argue that the ω and the ρ meson, and perhaps also the pion, are to be described by pole trajectories. As we have pointed out in the introduction and in reference 3, the pp-scattering experiments themselves are already sufficient if we assume the validity of some type of a Sommerfeld-Watson representation for $t\rightarrow\infty$. The difficulty with this representation is due to the fact that we do not know the behavior of $F_{\pm}(s,\lambda)$ for $\lambda\Rightarrow\lambda_r\pm i\infty$ with $\lambda_r<1$.

We would like to add a remark concerning the use of the possibility that the physical amplitude $F_0(s)$ is not determined by the interpolating function $F_+(s,\lambda)$ for $\lambda=0$. If $F_+(s,\lambda)$ has a vacuum pole trajectory $\lambda=\alpha_0(s)$ such that $\alpha_0(m^2(0))=0$ for some $m^2(0)<0$, then it is quite possible that this pole of $F_+(s,0)$ is not present in the physical amplitude $F_0(s)$ and hence does not give rise to any ghost problem.[12] On the other hand, if $\alpha_0(s)$ crosses the integer *two* such that $\alpha_0(m^2(2))=2$ for some point $s=m^2(2)$ in the second Riemann sheet,[13] then the arguments given in this paper show that the physical d-wave amplitude $F_2(s)$ must have a resonance pole.

We would like to thank Peter Freund for helpful discussions.

[10] R. Oehme, Phys. Rev. **121**, 1840 (1961).

[11] W. F. Baker, E. W. Jenkins, A. L. Read, G. Cocconi, V. T. Cocconi, and J. Orear, Phys. Rev. Letters **9**, 221 (1962). This paper contains further references.

[12] Another possible solution of this ghost problem has been proposed by M. Gell-Mann, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by T. Prentki (CERN, Geneva, 1962).

[13] C. Lovelace, Nuovo Cimento **25**, 730 (1962); V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **41**, 667 (1961) [translation: Soviet Phys.—JETP **14**, 478 (1962)]. G. F. Chew and S. C. Frautschi, reference 2.